VOLUME 56
NUMBER 6

WHOLE NO. 259
1944

Psychological review

Psychological Monographs

JOHN F. DASHIELL, *Editor*

The Effect of Special Construction of Test Items on Their Factor Composition

By Dora

CONSTANCE LOVELL

The University of Southern California

Published by

THE AMERICAN PSYCHOLOGICAL ASSOCIATION, INC.

Publications Office

NORTHWESTERN UNIVERSITY, EVANSTON, ILLINOIS

BF1
.P8

CONTENTS

	Page
I. The Problem	1
A. Background of the Study	1
B. The Plan of the Investigation	2
II. The Sources of Data	4
A. The Test Items	4
B. The Subjects and Items Used in the Factor Analyses	8
C. Correlation of the Data	9
III. The Factor Analyses	12
A. Factor Analysis of the Number-Series Items	12
B. Factor Analysis of the Figure-Analogies Items	13
IV. Interpretation of the Results of the Factor Analyses	17
A. Results from the Number-Series Items	17
B. Results from the Figure-Analogies Items	21
C. Relation of Factors to Difficulty of Items	24
V. Summary	25
Bibliography	26

THE LIBRARY OF
CONGRESS
SERIAL RECORD
AUG 7 1944
Copy 1

289078+
Aug. 7. 1943

THE EFFECT OF SPECIAL CONSTRUCTION OF TEST ITEMS ON THEIR FACTOR COMPOSITION[1]

CHAPTER I

THE PROBLEM

THE PURPOSE of this study was to determine whether the factor loading of test items could be influenced by constructing them according to certain hypotheses regarding the nature of factors of human ability.

A. BACKGROUND OF THE STUDY

Most tests of human ability are positively correlated. However, the varying sizes of the coefficients show that high performance on any one test is not equally associated with high scores on all the others. In a table of test intercorrelations, rough groupings can be made so that the association between the tests in any group is relatively large as compared to their associations with other tests in the battery. The high correlations between the tests in such a group indicate that there is some factor common to performance of the tasks involved. Any other group in the battery whose constituent tests are highly correlated involves a similar common factor. However, the two factors are not the same, because the tests of one group have relatively low correlations with those of the other. Any positive relationship existing between groups is an indication of overlapping factors.

The presence of underlying associations between tests is shown only crudely by such grouping according to the size of the inter-correlation coefficients. More refined techniques are necessary for accurate study of the factor pattern of a correlation table. The various systems of factor analysis were developed for this purpose. They provide the means of finding (1) the smallest number of factors by which a table of intercorrelations can be described and (2) the weight of each test in these factors.

The categories located in this manner may be described generally as functional unities contributing to performance in the tasks containing them. More specific identification of the operation each involves is made by studying the nature of the tasks which are weighted with them. Thus, the factor which has heavy loadings in tests of addition, multiplication, and division—and only slight loadings in verbal and performance tests—has been called the number factor. This and other factors identified as verbal, spatial, memory, and speed of perception have been located in many different tests and by several methods of factor analysis (1, 2, 3, 5, 12, 13, 14, 15, and 17).

Factor analysis studies have been handicapped greatly by the fact that most tests have very complex factor patterns (16). Because of this, identification of factors has had to remain very tentative in nature. In addition, no pure measures of the factors have been possible. Thurstone has commented regarding this problem:

> Now that several of the factors are somewhat better understood, it should also be

[1] This paper is based upon a dissertation submitted to the Department of Psychology, The University of Southern California, in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The writer wishes to express appreciation to Lt. Col. J. P. Guilford for his suggestion of the problem and for his invaluable help in supervision of the study.

expected that tests will be improved by increasing the saturation of the factor that each test is expected to measure and by decreasing the saturations of other factors that are measured by other tests. What we have called the "complexity" of each test should be reduced (15).

For decisive work on the nature of factors of human ability, then, tests of more simple structure are needed. The present research was an attempt to determine the efficacy of one possible method of securing more simple factor patterns.

B. THE PLAN OF THE INVESTIGATION

From the results of former studies, hypotheses have been set up concerning the characteristics which appear fundamental to the various factors of human ability. It is possible that one might secure tests with more simple factor patterns by constructing individual items so that they stressed the characteristics of one factor and minimized the characteristics of all others. If individual items could be formulated so as to be heavily loaded with a single factor, a group of such items might be expected to provide a relatively pure measure of the factor.

In the research here reported, attempt was made to formulate such items for two tests of human ability: number series and figure-analogies. Hypotheses formulated by J. P. Guilford concerning the type of item which might be expected to stress each of several factors were used in constructing the items.

Suggestions used in setting up figure-analogies items. With figure-analogies items, loadings have been obtained in factors named spatial ability, verbal ability, perceptual speed, deduction, and induction (9, p. 390). On the basis of former studies, Guilford has suggested types of figure-analogies items which might be expected to have heavy loadings in each of these factors. The following statements concerning these are quoted from his article:

> Perceptual speed will probably be most in evidence in a timed test when:
> (1) The figures are small or detailed. . . .
> (2) The figures are relatively complex. . . .
> (3) The change from A to B is slight. . . .
> (4) When in a multiple-choice response the correct figure is hard to discriminate from among its distractors, the rule or principle having been easy to apprehend. . . .
>
> The factor of spatial thinking will be most important when:
> (1) The principle of change depends upon a rotation . . . ;
> (2) The principle depends upon inversions . . . ; or
> (3) Upon the rearrangement of parts.
>
> The verbal factor might be expected to enter to some extent when the rule is complex or is easily and naturally verbalized, . . . This will also depend upon the individual, however, and his habits of depending or not depending upon verbalized methods. Here is a good example of vicarious functioning of unitary abilities.
>
> Deduction may be expected to enter most when:
> (1) There are fine distinctions to be made among several alternative rules all of which come close to fitting. . . . The relation in Item 9 could be conceived as one of size and change of position or as a whole-part relationship. The responses do not provide for the first rule but do for the second, therefore the second rule is correct.
> (2) There are fine distinctions to be made among alternative answers. . . .
>
> Finally, the inductive process, for which the test was primarily constructed, will have its greatest opportunity to reveal itself when:
> (1) All other factors are at a minimum, that is to say, when the figures are large and lacking in detail, no rotation or rearrangement of parts is required, and discriminations are simple.
> (2) The minimum essential for a correct response requires a correct apprehension of the rule, . . .
> (3) The same kind of principle is not repeated; a frequent change in kind of rule,

as in the number-series test, demands new inductions, and not simply the reproduction of already learned ones . . . (9, pp. 390-392).

Suggestions used in setting up number-series items. In tests of number-series items, factors have been found which have been identified as number, induction, deduction, and perceptual speed. Guilford formulated the following suggestions concerning these:

> The number factor is more important when the grasping of the principle and the selection of the answer depend much upon accurate computation. The inductive factor is more prominent when the computations required are very simple and computational slips are rare. The fact that most of the principles in number series are essentially numerical principles probably precludes an elimination of the number factor entirely. If a test is very much confined to a few kinds of principles, for example, if all of them are simple increments and decrements, the number factor should again come into greater prominence and perhaps the perceptual speed factor. On the other hand, if the principle changes very frequently and varies radically in kind from item to item, the induction factor should gain in importance. From this it would follow that a number-series test that is optimal for measuring inductive ability should change the kind of principle, at least slightly with every new item (9, p. 389).

Outline of procedure. This study involved (1) setting up a figure-analogies test and a number-series completion test, containing several items constructed according to each of the various methods suggested by Guilford; (2) administering the two tests to university students; (3) correlating, for each test, the responses to each item with those to every other item; and (4) making a factor analysis for each of the correlation matrices thus obtained, to see whether the several items constructed to stress any given factor had similar factor loadings.

For tests of human ability, the approach used in this investigation is a new one. Studies have been reported of factor analyses using the Thurstone method in which total tests have been set up to try out hypotheses concerning factors. None has been located in which responses on individual items so constructed were studied. In the field of personality tests, this has been done (*e.g.*, 10).

Chapter II

The Sources of Data

A. THE TEST ITEMS

Number-series completion items. Forty number-series completion items, formulated according to Guilford's suggestions, were divided into eight groups of five items, each of which aimed to stress a different factor. This plan is shown in the list below, which contains the methods used in trying to emphasize each factor and the eight groups of items.[2]

1. First group of items (number and perceptual speed factors emphasized by use of single principle and by avoiding complicated calculation).

								a	*b*	*c*	*d*	*e*
1)	12	17	22	27	32	37	42	37	45	47	52	57
2)	16	19	22	25	28	31	34	35	36	37	39	40
3)	10	14	18	22	26	30	34	38	40	42	44	46
4)	8	17	26	35	44	53	62	17	65	69	70	71
5)	3	10	17	24	31	38	45	60	56	55	54	52

2. Second group of items (perceptual speed factor emphasized by using single, simple principle and by using complicated answers).

6)	375	377	379	381	383	385	387	389	938	383	333	388
7)	$71\frac{1}{2}$	$74\frac{1}{2}$	$77\frac{1}{2}$	$80\frac{1}{2}$	$83\frac{1}{2}$	$86\frac{1}{2}$	$89\frac{1}{2}$	$90\frac{1}{2}$	$90\frac{1}{3}$	$92\frac{1}{2}$	$92\frac{2}{3}$	$92\frac{5}{8}$
8)	5^2	10^2	15^2	20^2	25^2	30^2	35^2	42^5	45^2	40^2	40^8	8^5
9)	139	149	159	169	179	189	199	209	219	229	292	200
10)*	1071	2072	3073	4074	5075	6076	7077	7078	8088	8708	9079	8078

3. Third group of items (induction factor emphasized by use of different principle for each item and by use of simple calculations).

11)	3	3	6	6	9	9	12	15	14	13	12	11
12)	$\frac{1}{2}$	1	2	4	8	16	32	44	58	60	62	64
13)	98	87	76	65	54	43	32	23	21	19	11	7
14)	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$1/_{12}$	$1/_{18}$	$1/_{21}$	$1/_{27}$	$1/_{30}$
15)	1	3	6	10	15	21	28	31	33	36	39	41

4. Fourth group of items (number factor emphasized by use of single principle and by using principle involving more complicated calculation).

16)*	8	11	15	20	26	33	41	49	50	82	85	90
17)	10	19	29	40	52	65	79	85	89	90	94	100
18)*	17	23	30	38	47	57	68	75	79	80	82	86
19)*	3	14	26	39	53	68	84	96	98	100	101	112
20)*	4	14	25	37	50	64	79	95	97	99	108	115

5. Fifth group of items (induction factor emphasized by varying principle and by using easy calculations).

21)	10	5	11	6	12	7	13	11	10	9	8	7
22)	4	8	5	9	6	10	7	13	9	12	11	10
23)*	80	40	44	22	26	13	17	21	8.5	6	4.5	4
24)	29	92	28	82	27	72	26	52	62	26	25	17
25)*	40	32	25	19	14	10	7	1	2	3	4	5

[2] Items used in the factor analysis are indicated by an asterisk. For selection of items, see page 9.

6. Sixth group of items (number factor emphasized by using only two principles and by dependence of answer upon accurate computation).

26)*	3	6	4	8	6	12	10	22	20	18	14	8
27)	3	9	6	18	15	45	42	126	127	128	129	130
28)*	1	3	7	21	25	75	79	238	237	225	221	156
29)*	2	4	9	18	23	46	51	102	101	75	56	53
30)	6	30	31	155	156	780	781	782	3905	395	78	781

7. Seventh group of items (induction factor emphasized by change of principle and by use of relatively simple calculations).

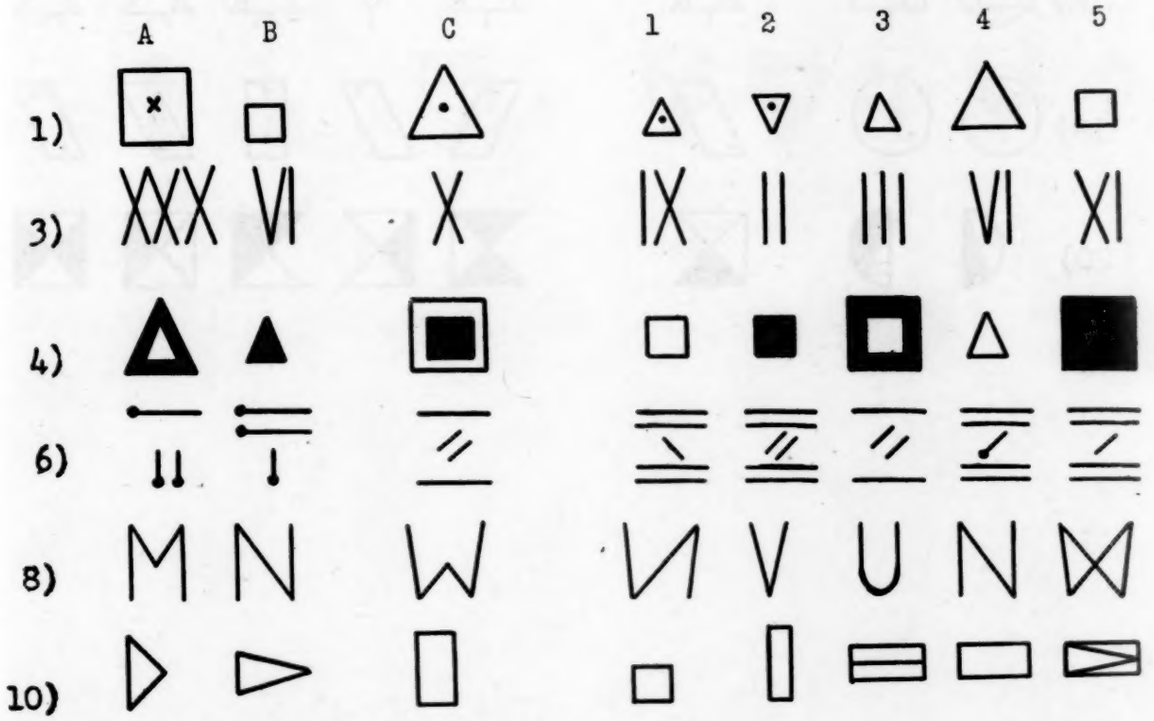
31)*	10	8	10	12	10	12	14	18	16	15	14	12
32)*	56	28	31	32	16	19	20	6	16	12	14	10
33)*	20	22	19	23	18	24	17	22	23	24	25	26
34)	1	4	10	22	46	94	190	382	360	350	255	198
35)*	98	89	78	87	76	67	56	45	54	65	74	76

8. Eighth group of items (deduction factor emphasized by using simple numbers and by using principles which might involve two answers with answer for only one given—e.g., the rule for item 36: add two, subtract three, multiply by two [not add two], add four).

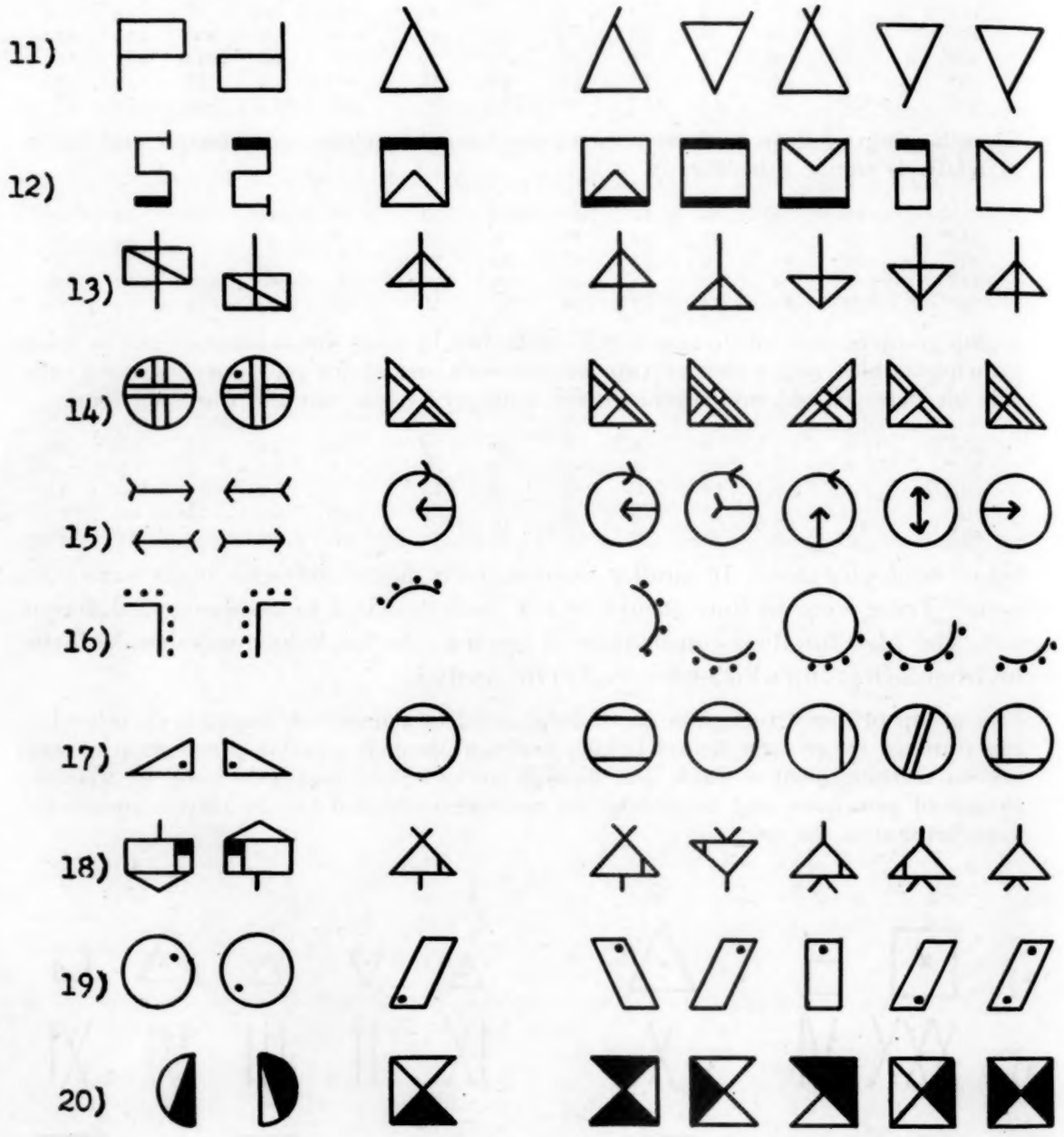
36)*	3	5	2	4	8	10	7	12	14	16	18	20
37)*	6	8	10	5	3	5	7	6	5	4	3	2
38)*	2	7	12	6	4	9	14	7	9	11	13	15
39)	2	3	4	16	4	5	6	16	20	24	26	27
40)	5	4	3	6	9	8	7	11	12	13	14	15

Figure-analogies items. In similar fashion, forty figure-analogies items were constructed. These were in four groups of ten, each designed to emphasize a different factor. The plan for their construction is given in the list below, together with the items from each group which were used in the analysis.

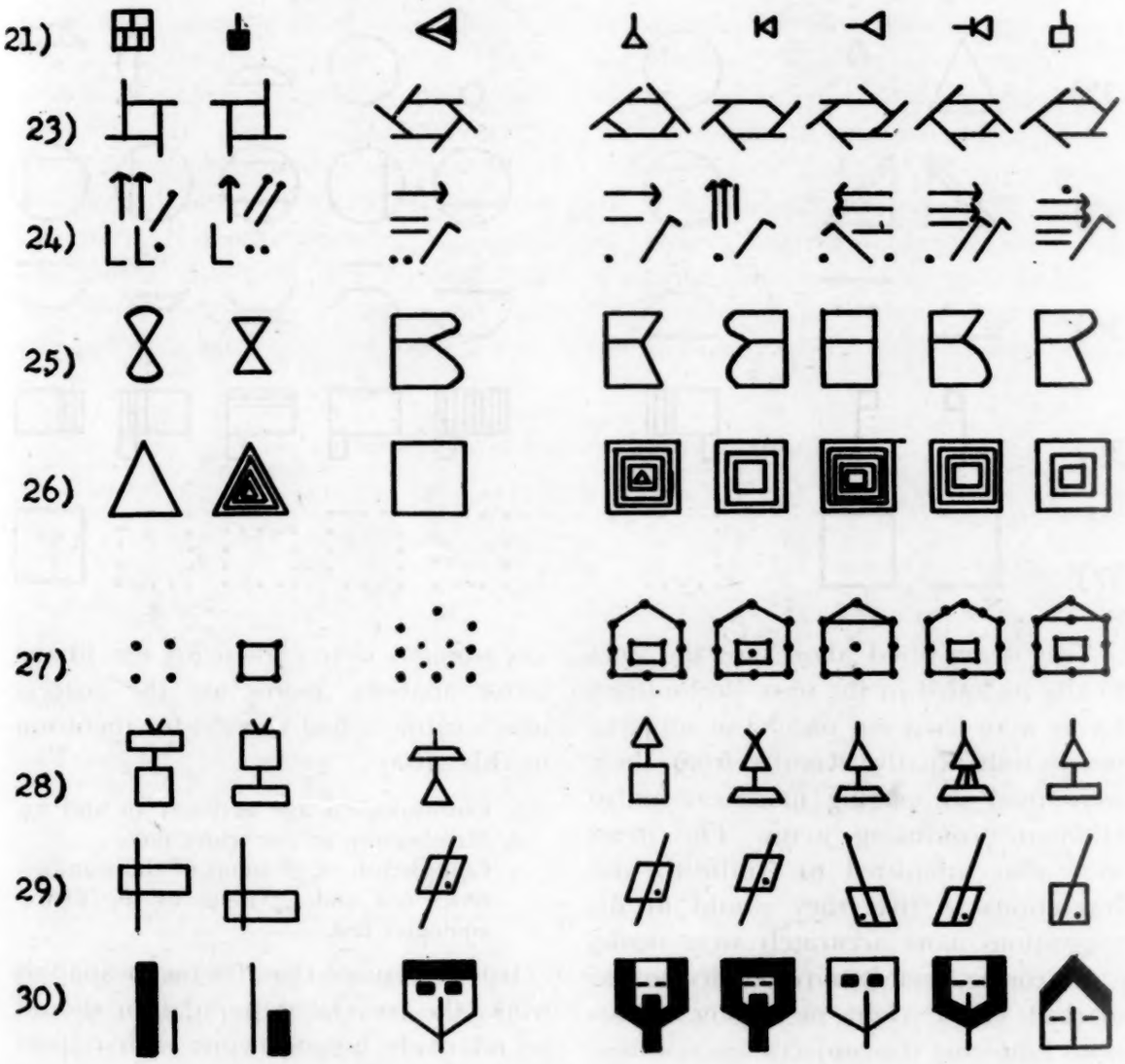
1. First group of items. Induction factor emphasized by minimizing emphasis on other factors (through use of large figures lacking in detail, through avoiding requirement of rotation of rearrangement of parts, and through use of simple discriminations), by frequent change of principles, and by making the minimum essential for the correct answer the apprehension of the rule.



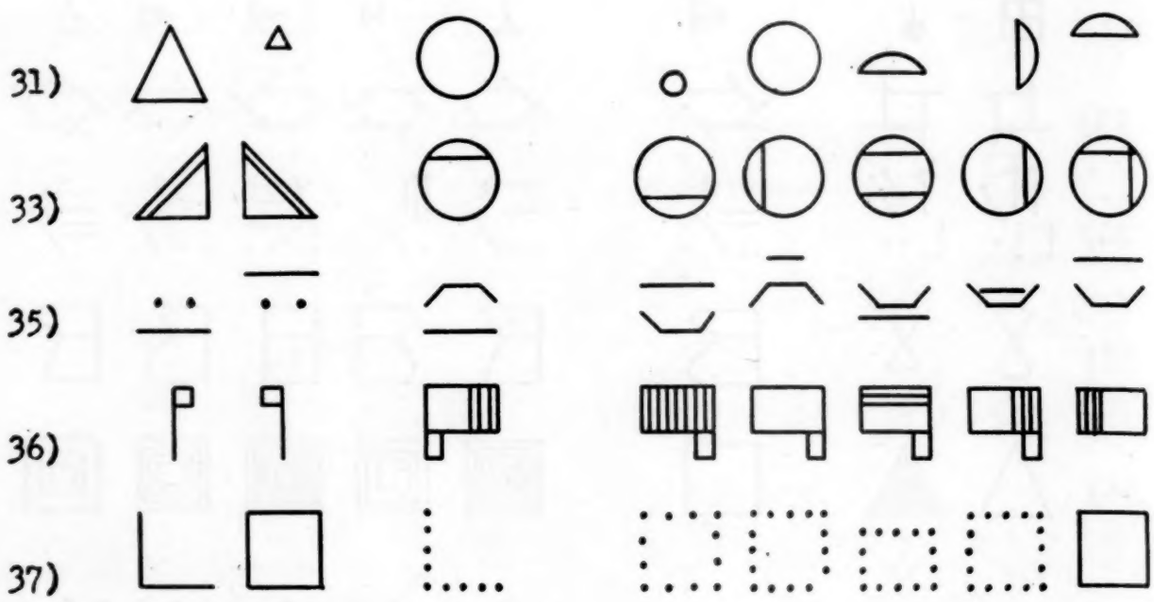
2. Second group of items. Spatial factor emphasized through use of rotation, inversions, and rearrangement of parts.



3. **Third group of items. Perceptual speed factor emphasized through use of small and detailed figures, through use of complex figures, and through use of slight changes in A and B as well as in multiple choice responses.**



4. Fourth group of items. Deduction factor emphasized by use of answers necessitating fine distinctions and by use of examples for which there might be two rules, with answer to satisfy only one.



The items listed above are the ones finally included in the tests. Preliminary forms were tried out on eleven subjects, tested individually. Results from these were used in making modifications to eliminate confusing items. The items were also submitted to Guilford, and corrections so that they would fit his suggestions more accurately were made.

In scoring, each item responded to was marked either right or wrong. Omissions following the subject's last response were marked as such. Those preceding the last response were scored wrong, on the assumption that the subject had tried them and had not been able to find an answer.

B. THE SUBJECTS AND ITEMS USED IN THE FACTOR ANALYSES

Selection of subjects. The tests were administered by the experimenter to eight elementary psychology classes. Altogether 543 subjects took the tests. The papers of 300 of these (187 men and 113 women) were chosen for use in the factor analyses. Below are the criteria which subjects had to meet for inclusion in this group:

1. Chronological age between 16 and 25.
2. Membership in the white race.
3. Completion of 38 items of the number-series test and 37 items of the figure-analogies test.

It is recognized that, for factor analysis work, the correlated population should be relatively homogeneous with respect to such variables as age, race, cultural background, and sex (9). The first two criteria for inclusion in the group of subjects were used to eliminate undue age and race heterogeneity. Since all these individuals were members of an undergraduate psychology class, they formed a relatively homogeneous group with respect to educational status and cultural background. Therefore, no additional criterion to control heterogeneity of cultural background was used. The factor of sex heterogeneity was tak-

en care of as indicated in the following section on the selection of items.

The third criterion was used in order to provide as many items as possible for the factor analyses. Only those items which all subjects had attempted could be correlated. Since some subjects did not finish the tests within the time limits allowed, the choice was, then, between (1) using all papers which met the qualifications of the first two criteria, with the result that many items would have been eliminated from the factor analyses because one or more persons had failed to reach them and (2) eliminating those subjects who had failed to get to the end of the test and correlating the results on all items for the remaining subjects. Actually, a middle path involving selection of subjects who had finished all but two items on the number-series test and three items on the figure-analogies test was used. The assumption involved in using this criterion was that what an individual did on any item (i.e., whether he got it right or wrong) was not affected by whether or not he attempted subsequent items and that, accordingly, the results on the subjects selected were not significantly different from those that would have been obtained if all subjects had been allowed to finish all items and the results for all subjects used in the correlations. The implications of this choice will be discussed in connection with the results obtained (p. 17).

Selection of items. For the total group (men and women together) the number of correct responses to each item was obtained. All items which were failed by more than 90 per cent or less than 10 per cent of the 300 subjects were eliminated.[3] For the remaining items, the proportion of passes for each sex was computed for each item. Only two items showed a very significant difference between the two sexes, and they were eliminated. The fact that the responses of men and women were so similar on the individual items was assurance that the factor loadings would not be influenced significantly by sex heterogeneity.

TABLE 1
Intercorrelations of number-series completion items

Item	10	16	18	19	20	23	25	26	28	29	31	32	33	35	36	37	38
10		.06	.20	.24	.19	.06	.04	−.04	.10	−.10	.02	.05	.15	.06	.07	.12	.22
16			.56	.50	.25	.10	.46	.31	.49	.44	.44	.25	.25	.06	.13	.30	.06
18				.66	.69	.37	.55	.25	.42	.38	.37	.35	.44	.22	.25	.39	.18
19					.47	.26	.46	.07	.13	.12	.35	.35	.47	.08	.12	.27	.17
20						.25	.43	.32	.32	.35	.11	.26	.42	−.04	.07	.33	.23
23							.21	.49	.53	.67	−.02	.44	.41	.06	.23	.20	.61
25								.22	.37	.33	.42	.51	.32	.05	.13	.32	.24
26									.49	.53	.13	.30	.40	.20	.21	.09	.37
28										.56	.29	.42	.45	.11	.20	.30	.44
29											.43	.42	.21	.18	.33	.27	.54
31												.43	.13	−.06	.30	.39	.31
32													.61	.19	.40	.45	.42
33														.30	.28	.36	.33
35															.14	.03	.07
36																.43	.46
37																	.59

C. CORRELATION OF THE DATA

By means of Thurstone's computing

[3] It was desirable that items with a more uneven split be eliminated since tetrachoric correlation coefficients were to be used.

TABLE 2
Intercorrelations of figure-analogies items

Item	1	3	4	6	8	10	11	12	13	14	15	16	17	18	19	20	21	23	24	25	26	27	28	29	30	31	33	35	36	37
1		03	−06	09	07	−03	09	07	−12	17	08	06	−06	02	01	10	14	12	09	15	−05	19	11	22	−04	−15	−08	−06	04	00
3			22	09	16	18	03	03	−06	08	14	04	20	06	05	17	25	25	28	26	22	11	11	−23	17	35	09	−05	13	−08
4				08	05	19	−06	06	23	−03	06	08	16	09	05	−05	04	12	07	03	−05	−06	05	−12	13	27	−08	08	−05	02
6					14	16	01	27	08	02	09	44	18	09	19	10	22	15	39	01	08	05	23	−22	25	06	−23	19	09	04
8						11	18	25	−02	−13	36	03	15	01	18	23	29	11	08	24	−02	02	05	−04	15	00	−09	−01	02	30
10							31	09	17	−06	19	10	12	−05	04	04	−12	05	00	16	00	−21	02	−08	04	26	−11	06	16	−06
11								04	15	−04	26	05	−07	11	03	21	19	−08	07	23	−04	06	−14	04	−03	01	13	07	−08	17
12									15	−06	−04	12	07	−06	00	04	27	22	15	12	06	−03	−05	−14	22	05	00	11	12	17
13										24	08	24	12	13	22	16	13	−06	27	18	05	15	02	−02	15	−19	01	16	05	05
14											06	−02	13	07	−01	01	−01	−01	16	03	03	05	13	10	−01	−16	−20	04	18	−09
15												10	00	11	17	30	12	−08	36	05	−05	16	00	−18	22	29	29	−03	05	29
16													−10	15	20	24	05	31	33	34	26	37	25	−29	28	16	12	08	06	31
17														08	15	12	10	01	14	15	05	−26	08	07	17	04	−30	12	30	−08
18															15	00	10	04	24	24	30	06	20	08	04	30	04	−02	−18	26
19																11	10	18	10	30	05	−02	05	−04	20	−03	13	−08	−03	14
20																	21	04	15	16	13	13	07	08	29	16	19	05	10	23
21																		09	25	17	13	17	19	01	27	13	−09	−04	01	13
23																			23	23	31	22	08	−09	13	21	−02	07	04	17
24																				38	18	10	17	−13	25	26	21	20	01	11
25																					11	10	04	−09	19	20	30	22	02	07
26																						22	11	−31	14	10	−16	00	27	28
27																							35	−18	08	05	−13	04	17	18
28																								11	27	12	05	10	08	00
29																									03	01	−03	12	−03	−05
30																										26	−10	08	32	11
31																											19	21	15	04
33																												06	−20	−05
35																													13	04
36																														15
37																														

Decimal points have been omitted in all correlation coefficients.

diagrams (4), tetrachoric correlation coefficients were then obtained of each number-series item with every other one, and of each figure-analogies item with every other one. The intercorrelations are given in table 1 and table 2.

Significance of number-series coefficients. Of the 136 number-series coefficients, seventy-nine were statistically significant (at the 5 per cent level).[4] Sixty-three of these were very significant (at the 1 per cent level). Only seven significant coefficients and one very significant coefficient would have been expected by chance alone. The large number actually found justified factor analysis of the data.

Significance of figure-analogies coefficients. Of the 435 figure-analogies coefficients, eighty were significant. Twenty-three of these were very significant. By chance alone, twenty-two significant and four very significant coefficients would have been expected. Since the number actually found was greater, factor analysis of the table seemed justified, despite the generally small size of the correlations.

[4] The significance of the correlation coefficients was determined from tables prepared by Guilford and Lyons (11).

TABLE 2
Intercorrelations of figure-analogies items

Item	1	3	4	6	8	10	11	12	13	14	15	16	17	18	19	20	21	23	24	25	26	27	28	29	30	31	33	35	36	37
1		03	−06	09	07	−03	09	07	−12	17	08	06	−06	02	01	10	14	12	09	15	−05	19	11	22	−04	−15	−08	−06	04	00
3			22	09	16	18	03	03	−06	08	14	04	20	06	05	17	25	25	28	26	22	11	11	−23	17	35	09	−05	13	−08
4				08	05	19	−06	06	23	−03	06	08	16	09	05	−05	04	12	07	03	−05	−06	05	−12	13	27	−08	08	−05	02
6					14	16	01	27	08	02	09	44	18	09	19	10	22	15	30	01	08	05	23	−22	25	06	−23	19	09	04
8						11	18	25	−02	−13	36	03	15	01	18	23	29	11	08	24	−02	02	05	−04	15	00	−09	−01	02	30
10							31	09	17	−06	19	10	12	−05	04	04	−12	05	00	16	00	−21	02	−08	04	26	−11	06	16	−06
11								04	15	−04	26	05	−07	11	03	21	19	−08	07	23	−04	06	−14	04	−03	01	13	07	−08	17
12									15	−06	−04	12	07	−06	00	04	27	22	15	12	06	−03	−05	−14	22	05	00	11	12	17
13										24	08	24	12	13	22	16	13	−06	27	18	05	15	02	−02	15	−19	01	16	05	05
14											06	−02	13	07	−01	01	−01	−01	16	03	03	05	13	10	−01	−16	−20	04	18	−09
15												10	00	11	17	30	12	−08	36	05	−05	16	00	−18	22	29	29	−03	05	29
16													−10	15	20	24	05	31	33	34	26	37	25	−29	28	16	12	08	06	31
17														08	15	12	10	01	14	15	05	−26	08	07	17	04	−30	12	30	−08
18															15	00	10	04	24	24	30	06	20	08	04	30	04	−02	−18	26
19																11	10	18	10	30	05	−02	05	−04	20	−03	13	−08	−03	14
20																	21	04	15	16	13	13	07	08	29	16	19	05	10	23
21																		09	25	17	13	17	19	01	27	13	−09	−04	01	13
23																			23	23	31	22	08	−09	13	21	−02	07	04	17
24																				38	18	10	17	−13	25	26	21	20	01	11
25																					11	10	04	−09	19	20	30	22	02	07
26																						22	11	−31	14	10	−16	00	27	28
27																							35	−18	08	05	−13	04	17	18
28																								11	27	12	05	10	08	00
29																									03	01	−03	12	−03	−05
30																										26	−10	08	32	11
31																											19	21	15	04
33																												06	−20	−05
35																													13	04
36																														15
37																														

Decimal points have been omitted in all correlation coefficients.

diagrams (4), tetrachoric correlation coefficients were then obtained of each number-series item with every other one, and of each figure-analogies item with every other one. The intercorrelations are given in table 1 and table 2.

Significance of number-series coefficients. Of the 136 number-series coefficients, seventy-nine were statistically significant (at the 5 per cent level).[4] Sixty-three of these were very significant (at the 1 per cent level). Only seven significant coefficients and one very significant coefficient would have been expected by chance alone. The large number actually found justified factor analysis of the data.

Significance of figure-analogies coefficients. Of the 435 figure-analogies coefficients, eighty were significant. Twenty-three of these were very significant. By chance alone, twenty-two significant and four very significant coefficients would have been expected. Since the number actually found was greater, factor analysis of the table seemed justified, despite the generally small size of the correlations.

[4] The significance of the correlation coefficients was determined from tables prepared by Guilford and Lyons (11).

Chapter III

The Factor Analyses

A. FACTOR ANALYSIS OF THE NUMBER-SERIES ITEMS

EXTRACTION *of centroid factors.* The Thurstone method of factor analysis was used, and extraction of centroid factors was made according to the procedure outlined by Guilford (8, pp. 478-488).[5] Five criteria were used in determining when to stop extraction:

1. Coombs' criterion (6).
2. Tucker's criterion (revised).
3. Range of factor loadings.
4. Maximum contribution to communality.
5. Comparison of the standard deviation of the residuals of each factor with the standard error of the average coefficient in the original correlation matrix.

According to the first four criteria, there were five significant factors in the number-series data. The fifth criterion suggested that only three factors should be extracted, but it was considered too crude to be decisive. Accordingly, five factors were used in rotation.

After the first extraction of five factors, the communality for each number-series item was obtained by computing the sum of the squares of the factor loadings for each item. The figures thus computed, when compared with the communalities used at the beginning of the analysis, revealed one difference of .17 and six others of .10 or more. These discrepancies were considered too large to be tolerated. With the communalities obtained from the factor loadings of the first analysis, a second set of extractions was made. The results of this analysis showed two discrepancies of .06 between the communalities used at the beginning and those found at the end. With the new communalities, a third set of extractions was made. One discrepancy of .06 was found with this, but all the rest of the differences were .03 or less. The fact that the size of the largest discrepancy was no smaller after the third analysis than after the second suggested that stability in the communalities had been reached and that further approximation of the communalities for subsequent extractions would not give smaller discrepancies. The centroid loadings and communalities obtained from the final series of extractions are given in table 3.

Rotation of number-series axes. Rotation of the axes was made graphically according to the procedure outlined by Guilford (8, pp. 489-491 and 502-507). The aim in rotation was to maximize the number of vanishing entries and to minimize the number and size of negative entries.[6] The procedure was repeated until further rotation yielded no improvement according to these criteria. For the number-series items, no negative entries of greater than −.2 were present in the final factor loadings. The factor loadings and communalities from the final rotation are given in table 4.

[5] Before the factor analysis was begun, the items were renumbered in chance order. After the rotations had been completed, the items were given their original numbers and they are used throughout this report.

[6] Loadings of +.3 and above or of −.3 and below were considered as significant in naming factors; those from .20 to .29 and from −.20 to −.29 were considered as greater than zero but too small to be important in identifying the factors; and those between .19 and −.19 were regarded as vanishing entries.

TABLE 3
Centroid factor loadings and obtained communalities for number-series completion items*
(Third Set of Extractions)

Item	I	II	III	IV	V
10	166 (028)	120 (014)	−165 (027)	107 (011)	−159 (025)
16	562 (316)	335 (112)	244 (060)	−196 (038)	278 (077)
18	765 (585)	448 (201)	186 (035)	217 (047)	−105 (011)
19	575 (331)	502 (252)	−085 (007)	311 (097)	−014 (000)
20	556 (309)	282 (080)	233 (054)	263 (069)	−247 (061)
23	594 (353)	−484 (234)	205 (042)	107 (011)	−343 (118)
25	591 (349)	272 (074)	076 (006)	017 (000)	186 (035)
26	518 (268)	−378 (143)	305 (093)	009 (000)	118 (014)
28	659 (434)	−176 (031)	262 (069)	−100 (010)	141 (020)
29	688 (473)	−316 (100)	350 (123)	−318 (101)	−034 (001)
31	494 (244)	249 (062)	−140 (020)	−452 (204)	187 (035)
32	690 (476)	−134 (018)	−245 (060)	060 (004)	232 (054)
33	673 (453)	−149 (022)	−227 (052)	469 (220)	220 (048)
35	192 (037)	−184 (034)	−059 (003)	182 (033)	098 (010)
36	439 (193)	−167 (028)	−333 (111)	−179 (032)	−062 (004)
37	578 (334)	097 (009)	−348 (121)	−267 (071)	−139 (019)
38	638 (407)	−319 (102)	−258 (067)	−230 (053)	−375 (141)

* The communality obtained for each item is given in parentheses following the factor loading. Decimal points have been omitted preceding each factor loading and communality in the table.

B. FACTOR ANALYSIS OF THE FIGURE-ANALOGIES ITEMS

Extraction of centroid factors. The procedure used in extracting centroid factors for these items was the same as that used in connection with the number-series items.

For these data, the criteria of when to stop extraction were not in agreement. Tucker's criterion did not give a clear-cut indication of when to stop extracting. A second criterion (that of the standard deviation of the residuals) pointed to the extraction of only four factors, but it was a rough method of determination. Coombs' criterion suggested that there were no more than six significant factors, but the two other criteria indicated

TABLE 4
Rotated factor loadings and communalities for number-series items*

Item	I	II	III	IV	V
10	123 (015)	−087 (008)	289 (083)	−011 (000)	019 (000)
16	588 (346)	261 (068)	−057 (003)	431 (186)	011 (000)
18	517 (267)	038 (001)	475 (226)	620 (384)	003 (000)
19	479 (230)	052 (003)	552 (305)	335 (112)	−199 (040)
20	249 (062)	−089 (008)	424 (180)	566 (320)	056 (003)
23	−159 (025)	143 (020)	335 (112)	403 (163)	662 (438)
25	478 (229)	264 (070)	204 (042)	352 (124)	002 (000)
26	−037 (001)	425 (181)	038 (001)	415 (172)	404 (164)
28	234 (055)	405 (164)	056 (003)	441 (195)	384 (147)
29	224 (050)	297 (088)	−065 (004)	463 (214)	665 (442)
31	696 (485)	198 (039)	−050 (003)	003 (000)	196 (039)
32	318 (101)	539 (291)	389 (151)	066 (004)	254 (065)
33	092 (009)	577 (333)	647 (419)	177 (031)	066 (004)
35	−088 (008)	257 (066)	192 (037)	042 (002)	067 (005)
36	266 (071)	195 (038)	254 (064)	−163 (026)	409 (167)
37	546 (298)	069 (005)	312 (097)	−091 (008)	384 (148)
38	236 (056)	066 (004)	373 (139)	−029 (001)	754 (569)

* The communality obtained for each item is given in parentheses following the factor loading. Decimal points have been omitted preceding each factor loading and communality in the table.

TABLE 5
Centroid factor loadings and obtained communalities for figure-analogies items*

Item	I	II	III	IV	V	VI	VII	VIII	IX
1	102 (010)	142 (020)	−126 (016)	−179 (032)	197 (039)	119 (014)	087 (008)	−110 (012)	186 (035)
3	426 (181)	−232 (054)	166 (028)	098 (010)	−162 (026)	−058 (003)	122 (015)	−297 (088)	336 (113)
4	212 (045)	−137 (019)	123 (015)	258 (067)	−216 (047)	−135 (018)	−081 (007)	128 (016)	096 (009)
6	440 (194)	119 (014)	−124 (015)	101 (010)	260 (068)	−366 (134)	−108 (012)	277 (077)	175 (031)
8	345 (119)	−134 (018)	−452 (204)	−246 (061)	−112 (013)	−055 (003)	−079 (006)	−101 (010)	170 (029)
10	229 (052)	−171 (029)	−129 (017)	245 (060)	−329 (108)	−069 (005)	255 (065)	242 (059)	−113 (013)
11	222 (049)	063 (004)	−237 (056)	−223 (050)	−291 (085)	247 (061)	143 (020)	185 (034)	−111 (012)
12	297 (088)	−066 (004)	−187 (035)	−051 (003)	173 (030)	−305 (093)	−109 (012)	109 (012)	−110 (012)
13	316 (100)	310 (096)	033 (001)	069 (005)	−053 (003)	092 (008)	173 (030)	376 (141)	−086 (007)
14	074 (005)	221 (049)	063 (004)	223 (050)	126 (016)	274 (075)	195 (038)	086 (007)	191 (036)
15	435 (189)	−072 (005)	−208 (043)	−230 (053)	−300 (090)	212 (045)	−175 (031)	110 (012)	194 (038)
16	561 (315)	052 (003)	223 (050)	−176 (031)	284 (081)	−165 (027)	071 (005)	185 (034)	−200 (040)
17	209 (044)	083 (007)	−208 (043)	476 (227)	−135 (018)	−088 (008)	129 (017)	−140 (020)	112 (013)
18	292 (085)	149 (022)	231 (053)	055 (003)	−096 (009)	245 (060)	−209 (044)	−071 (005)	−101 (010)
19	310 (096)	176 (031)	−083 (007)	−088 (008)	−051 (003)	−121 (015)	118 (014)	−103 (011)	−039 (002)
20	408 (166)	107 (011)	−214 (046)	−118 (014)	−054 (003)	139 (019)	043 (002)	−142 (020)	−094 (009)
21	393 (154)	065 (004)	−188 (035)	−081 (007)	111 (012)	049 (002)	−155 (024)	−185 (034)	170 (029)
23	368 (135)	−222 (049)	213 (045)	−113 (013)	176 (031)	−262 (069)	113 (013)	−181 (033)	−072 (005)
24	599 (359)	243 (059)	217 (047)	−027 (001)	−035 (001)	−093 (009)	−165 (027)	122 (015)	157 (025)
25	537 (288)	258 (067)	170 (029)	−209 (044)	−251 (063)	−220 (048)	319 (102)	−257 (066)	−133 (018)
26	347 (120)	−231 (053)	217 (047)	040 (002)	279 (078)	167 (028)	146 (021)	−118 (014)	−203 (041)
27	299 (089)	−070 (005)	248 (062)	−211 (045)	419 (176)	364 (132)	050 (003)	−041 (002)	099 (010)
28	311 (097)	194 (038)	160 (026)	198 (039)	272 (074)	043 (002)	−226 (051)	−238 (057)	138 (019)
29	−178 (032)	413 (171)	−205 (042)	161 (026)	−053 (003)	200 (040)	−140 (020)	−280 (078)	−193 (037)
30	471 (222)	021 (000)	−163 (027)	179 (032)	111 (012)	−072 (005)	−124 (015)	−145 (021)	−111 (012)
31	384 (147)	−241 (058)	188 (035)	182 (033)	−331 (110)	−079 (006)	−352 (124)	−232 (054)	−186 (035)
33	031 (001)	153 (023)	242 (059)	−447 (200)	−452 (204)	−058 (003)	−124 (015)	−086 (007)	−190 (036)
35	200 (040)	117 (014)	092 (008)	166 (028)	−059 (003)	−084 (007)	−098 (010)	110 (012)	−168 (028)
36	256 (066)	−208 (043)	−159 (025)	349 (122)	252 (064)	061 (004)	195 (038)	−087 (008)	−134 (018)
37	340 (116)	−180 (032)	−155 (024)	−268 (072)	099 (010)	258 (067)	−158 (025)	080 (006)	−305 (093)

* The communality obtained for each item is given in parentheses following the factor loading. Decimal points have been omitted preceding each factor loading and communality in the table.

TABLE 6
Rotated factor loadings and communalities for figure-analogies items*

Item	I	II	III	IV	V	VI	VII	VIII	IX
1	−002 (000)	087 (008)	080 (006)	−111 (012)	172 (030)	−103 (011)	173 (030)	189 (036)	−231 (053)
3	−081 (007)	−050 (003)	−067 (005)	067 (005)	477 (227)	429 (184)	251 (063)	063 (004)	155 (024)
4	072 (005)	112 (013)	−096 (009)	032 (001)	065 (004)	310 (096)	−028 (001)	−099 (010)	321 (103)
6	−034 (001)	643 (413)	−081 (007)	−028 (001)	069 (005)	−010 (000)	128 (017)	195 (038)	275 (076)
8	046 (002)	014 (000)	−024 (001)	−029 (001)	171 (029)	113 (013)	006 (000)	642 (412)	081 (007)
10	345 (119)	−066 (004)	−114 (013)	129 (017)	165 (027)	004 (000)	−103 (011)	−003 (000)	467 (218)
11	502 (252)	−083 (007)	−005 (000)	118 (014)	077 (006)	−038 (001)	−099 (010)	279 (078)	−061 (004)
12	−091 (008)	285 (081)	−072 (005)	169 (029)	−086 (007)	−077 (006)	078 (006)	301 (090)	237 (056)
13	473 (224)	342 (117)	046 (002)	147 (022)	092 (008)	−101 (010)	015 (000)	−092 (008)	044 (002)
14	237 (056)	130 (017)	167 (028)	−214 (046)	195 (038)	−068 (005)	199 (039)	−228 (052)	−053 (003)
15	380 (145)	063 (004)	−041 (002)	−012 (000)	081 (007)	367 (135)	012 (000)	460 (212)	−033 (001)
16	098 (010)	429 (184)	−137 (019)	477 (227)	−008 (000)	−043 (002)	370 (137)	081 (007)	017 (000)
17	027 (001)	057 (003)	301 (091)	−092 (009)	333 (111)	056 (003)	−034 (001)	−022 (001)	421 (177)
18	192 (037)	088 (008)	254 (064)	187 (035)	−077 (006)	329 (108)	134 (018)	−069 (005)	−109 (012)
19	042 (002)	152 (023)	110 (012)	240 (057)	238 (057)	−022 (001)	001 (000)	181 (033)	006 (000)
20	203 (041)	029 (001)	232 (054)	196 (038)	151 (023)	033 (001)	128 (016)	343 (117)	−017 (000)
21	−030 (001)	183 (033)	191 (037)	−017 (000)	134 (018)	170 (029)	209 (044)	375 (140)	−030 (001)
23	−232 (054)	074 (006)	−169 (028)	387 (150)	147 (022)	076 (006)	334 (111)	−070 (005)	110 (012)
24	166 (028)	535 (287)	035 (001)	224 (050)	155 (024)	358 (128)	137 (019)	072 (005)	−022 (001)
25	075 (006)	138 (019)	091 (008)	642 (412)	505 (255)	089 (008)	−008 (001)	108 (012)	−079 (006)
26	040 (002)	−077 (006)	022 (001)	278 (078)	−011 (000)	012 (000)	559 (313)	−048 (002)	077 (006)
27	098 (010)	075 (006)	−033 (001)	032 (001)	000 (000)	054 (003)	634 (401)	024 (001)	−320 (103)
28	−169 (029)	304 (093)	337 (114)	−010 (000)	053 (003)	238 (057)	323 (105)	−048 (002)	−035 (001)
29	−004 (000)	−085 (007)	600 (360)	−061 (004)	−079 (006)	−100 (010)	−209 (044)	−020 (000)	−126 (016)
30	−017 (000)	198 (039)	286 (082)	174 (030)	039 (002)	105 (011)	220 (048)	242 (059)	279 (078)
31	−054 (003)	−085 (007)	137 (019)	354 (126)	−098 (010)	589 (347)	031 (001)	064 (004)	295 (087)
33	078 (006)	−081 (007)	−081 (007)	447 (200)	−023 (001)	257 (066)	−356 (127)	072 (005)	−363 (132)
35	099 (010)	202 (041)	110 (012)	193 (037)	−079 (006)	090 (008)	−032 (001)	−091 (008)	164 (027)
36	021 (001)	−066 (004)	167 (028)	018 (000)	071 (005)	−150 (023)	396 (156)	025 (001)	409 (167)
37	261 (068)	−056 (003)	013 (000)	227 (052)	−298 (089)	022 (001)	280 (079)	392 (153)	−031 (001)

* The communality obtained for each item is given in parentheses following the factor loading. Decimal points have been omitted preceding each factor loading and communality in the table.

that there were nine. It was decided to use nine factors in rotation. If there were actually fewer factors, the results would not be hurt by using the larger number. The centroid factor loadings and obtained communalities for the figure-analogies items are shown in table 5. Despite some large discrepancies between estimated and obtained communalities, successive extractions using new approximations of the communality were not made because of the large number of variables.

Rotation of figure-analogies axes. The procedure used in rotating axes for these data was also the same as that used in connection with the number-series items. In this part of the study it was impossible to eliminate all significantly large negative entries. Three items of −.3 or more were present after the last rotation had been made. The existence of such entries was not unexpected, however, because of the fact that some of the original correlations were negative. The factor loadings and communalities from the final rotation are given in table 6.

Chapter IV

Interpretation of the Results of the Factor Analyses

A. Results from number-series items

In the construction of the number-series items, hypotheses concerning the nature of four factors (perceptual speed, number, induction, and deduction) were used, with the intention of securing four groups of items, each heavily loaded with one of the four factors.

"Perceptual speed" items. When the final selection of items for the analysis was made, only one item from those intended to emphasize perceptual speed was used. The other items had been passed by too large a percentage of the subjects (more than 90 per cent) to be used in computing correlation coefficients. Inasmuch as it takes more than one item to produce a common factor, no factor of perceptual speed was expected in the analysis. Actually, this one item (10) had no significant loading in any factor.

It is possible that the procedure used in making this study is inadequate for revealing loadings in the perceptual speed factor. The total time allowed for the test was limited, but no specific controls were placed on the time spent on individual items. It seems reasonable to assume that persons low in perceptual speed would, within the total time limit, simply spend more time on any items heavily loaded with perceptual speed than other persons. Additional time per item would enable them to get individual items correct. Differences in perceptual speed would, thus, tend to show themselves in terms of number of items completed, rather than in terms of correct answers on "perceptual speed" items. In this study, individuals who did not complete a certain number of items were not used as subjects. If those persons low in perceptual speed finished fewer items, some variance in perceptual speed was lost through this procedure. However, if perceptual speed variance showed itself only in terms of number of items completed (and not in terms of accuracy of response on certain items), one would not expect to find specific items loaded with a common factor of perceptual speed, even if all individuals had been used. It may be that variance in perceptual speed cannot be found in single items of a test unless the time of exposure of each item is experimentally controlled.

It may be stated, further, that some variance in the other factors may have been lost through the procedure of eliminating subjects. Individuals low in "number" ability might be expected to take more time than the average on "number" items and thus to finish fewer items in the total test. If this were the case, however, one might expect that any grouping of "number" items (or of other types of items) found in this study would be even more clear-cut if items had been individually timed and the total variance retained.

"Number items. Included in the analysis were seven items which had been constructed with the idea of emphasizing the number factor. The loadings for these items were distributed as follows:

Factor	*Loading*			
	.30 or above	.20 to .29	.19 to —.19	—.20 to —.29
I	3	3	1	0
II	3	1	3	0
III	3	0	4	0
IV	7	0	0	0
V	3	0	3	1

All these "number" items had loadings

of .30 or above on only one factor—Factor IV. For that reason it has been tentatively identified as a number factor. That all the "number" items had relatively high loadings on one factor and on only one gives support to the idea that the rules used in constructing them involved various aspects of some functional unity.

However, construction of these items according to the hypotheses proposed by Guilford did not provide complete control over the loadings. Four of the "number" items (16, 19, 26, 29) had higher loadings in some other factor than in Factor IV, and two of the items designed to emphasize other factors (23 and 25) had as heavy loadings in Factor IV as the "number" items. (The loadings of 23 and 25 on Factor IV were, however, not their highest loadings.)

Analysis of the items significantly loaded with Factor IV revealed that they appeared to involve two types of rule: (1) a principle involving addition or subtraction of a series which increases or decreases by units of one, such as +1, +2, +3, etc.; and (2) a principle with only two elements, such as ×2, −2. Both of these appear to be simpler than the rules used in the other items. It is logical to suspect that individual differences in the items with the simplest principles would be influenced more by differences in ability to perform simple numerical tasks than by ability to figure out principles. The present interpretation of the number factor is that it deals with such ability to perform simple tasks, and the findings of this study are in line with this point of view.

Despite the fact that use of the hypotheses did not give complete control over the factors involved in the "number" items, the ability to construct items with significant loadings in a factor is of sufficient importance to merit further work with this technique. The logical picture of the items loaded with the "number" factor further suggests that, with more careful construction of items, even more accurate prediction of the factor pattern of "number" items may be made.

"Induction" items. For the six "induction" items used in the analysis, factor loadings were distributed as follows:

Factor	*Loading*			
	.30 or above	.20 to .29	.19 to −.19	−.20 to −.29
I	3	0	3	0
II	2	3	1	0
III	3	1	2	0
IV	2	0	4	0
V	1	2	3	0

On no factor did all these items have significant loadings. Use of the induction hypotheses, therefore, was not as successful as the use of the hypotheses regarding the number factor. However, despite the failure of a clear-cut induction factor to appear, it was thought possible that one of the factors found might qualify as inductive in nature—even though all the "induction" items did not have significant loadings on it. For Factor II, all the "induction" items except one had loadings greater than zero. Study of the loadings for all items on this factor indicated that two of the "induction" items had the largest of the five significant loadings. The other three items with significant loadings were "number" items. Two of them involved changes of principle from the items before them. The only change in the other was a change of the actual numbers used in applying the principle found in the previous item. Of the five items with loadings between .20 and .29, three were "induction" items. One of the others was a "number"

item, and the only change it contained was a change of the actual numbers used. The remaining item in this group was a "deduction" item. The loading of items of this type probably depends on the relative importance of figuring out and applying the principle involved. Thus, some "deduction" items might possibly have loadings in induction and others, not. With one exception, other items with vanishing loadings in this factor were all ones stressing computation and free from change of principle.

On Factor III, the "induction" items had three of the eight loadings of .3 and above. Two other items with high loadings were from the "deduction" group. Inspection of the test blank revealed that they also involved changes of principle from the one used in the previous item. However, the other three items with significant loadings—"number" items—definitely contain the same kind of principle that was used in the preceding item. The only change involved is one connected with the actual numbers used. In addition, several of the items with zero loadings in this factor involve very definite changes of principle (26, 28, 29, 31). Obviously, this idea is not a key to the nature of the factor. Just what the factor may be is difficult to determine from the data available. The factor is not connected with the difficulty of the items, since those with the same proportions of passes appear with both significant and zero loadings. Also, it is not connected with the length of the series in the principle, for rules of various lengths appear with both high and vanishing loadings. It seems unlikely that the factor involves ability to verbalize, for items which appear very similar are distributed without any order from the top to the bottom of the list (*e.g.*, +11, +12, +13 with a loading of .552 and +3, +4, +5 with a loading of −.057).

In summary it may be said that no induction factor is shown in clear-cut fashion. However, of the two factors in which "induction" items have few zero loadings, Factor II shows some relationship to previously found factors identified as inductive. It is, therefore, tentatively named an induction factor.

"Deduction" items. For the three "deduction" items, loadings in the five factors were as follows:

Factor	*Loading*			
	.30 or above	.20. to .29	.19 to −.19	−.20 to −.29
I	1	2	0	0
II	0	1	2	0
III	2	1	0	0
IV	0	0	3	0
V	3	0	0	0

Three items are too few to give decisive evidence regarding ability to influence factor loadings by special construction of test items. However, all three of these items had significant loadings in Factor V. Two of them had their highest loadings in this factor. If these items were taken as a sample of the kind which would be constructed according to the "deduction" hypotheses, it is this factor which would be considered as a deduction factor.

Of the seven items with loadings of .30 or above on this factor, three are the "deduction" items. Three "number" items and one "induction" item also have such loadings. The principle involved in constructing the "deduction" items consisted of setting up series for which two rules might be formulated and requiring choice between these rules by giving the answer for only one of them. When the items were checked again, it was found that all those items

with loadings of .30 or above on this factor did contain the possibility of choice between principles. In the "deduction" items, the choice between rules could be made only on the basis of the one correct answer given. For the "induction" item and the three "number" items there was some leeway as to principle, but in each of these cases, the principle required for the correct answer was repeated within the item itself and deduction as to the correct rule could be made without reference to the answers. (For example, item 28 consisted of these two possibilities. +2, +4, +14, +4, +50, +4; ×3, +4, ×3, +4, ×3, +4.) None of the items with zero loadings involved any evident choice of this sort between two principles.

With the items analyzed in this way, a rather clear-cut "deduction" factor seemed obvious. It fits in with the hypothesis used in constructing the items and with previous descriptions of deductive factors.

Other factors. Although only four factors were intended in this study, and only three were expected after the elimination of the "perceptual speed" items, actually five were obtained. Factor III has been discussed previously. The nature of Factor I is not evident from an inspection of the principles of the various items which have heavy loadings in it. Inasmuch as seven out of the seventeen items have a significant loading in this factor, it is of some importance, but its distinguishing characteristics are too subtle to be singled out from study of the items alone. It is possible that the two factors whose nature is not obvious are of a type which would be found only in number-series completion items. Surely it is logical to think that items so similar in form and material may involve common mental processes not present in other tasks. Thus, the finding of more—or different—common factors in an analysis of items of a single test than in an analysis of several different tests should not be a surprising occurrence.

Summary and general discussion of results. For the number-series material, three of the obtained factors were tentatively identified as a number factor, a deduction factor, and an induction factor. These were three which the use of Guilford's hypotheses in the construction of items was intended to emphasize. To a certain extent, then, the results on these items supported the descriptions of the factors used in the hypotheses, and also the procedure of using the hypotheses to control the loadings of the items.

Though the general results on this part of the study were positive, there were enough findings which threw doubt on the general conclusion to call for further improvement of this method. For example, some of the "number" items had high loadings on other factors, and some of the "induction" items did not have as high loadings on the "induction" factor as expected if the hypotheses were completely substantiated.

Two reasons may exist for the discrepancies found. In the first place, more careful construction of items according to the hypotheses may be necessary. For example, "number" items could have high loadings on the "induction" factor without invalidating the hypotheses because new principles, as well as the necessity for accurate computation, were involved in them. An item constructed so as to fit the hypotheses for more than one factor could not be expected to show clear-cut differences in an analysis. The fact that items may easily have the characteristics of more than one factor

makes the problem a difficult one. For instance, the induction and deduction factors necessarily involve working with materials of some sort, so it may be impossible to eliminate from such items a loading in a factor of ability to deal with the material. Items will probably have to be selected which have some, but relatively little loading in a second factor.

When the material for the present tests was developed, attempt was made to prevent an item from fitting the hypotheses for more than one factor. Where this seemed impossible, effort was made—on the basis of logical considerations only—to minimize the importance of the second factor. However, this study has revealed clearly that such procedure is not an adequate method of determining the relative importance of the various processes required in solving an item. The procedure of getting reports of what individuals actually do with the items appears a much more satisfactory way of dealing with this problem. Even if such accounts were not altogether accurate, they might furnish valuable cues as to the process used in dealing with the items. This method of approach seems very clearly to be the next step for research in this field.

In the second place, discrepancies found between the designed and actual factor loadings may have resulted from the fact that the hypotheses need modification to fit more accurately the actual nature of the factors. Results from the use of these items should be studied in relation to other types of task, whose factor patterns are already known. The factors found in these specially formulated items might then be more clearly identified in terms of the descriptions of previously found factors. Such procedure might even suggest interpretations for factors that, by themselves, appear to have no meaning at all.

B. RESULTS FROM FIGURE-ANALOGIES ITEMS

The figure-analogies item were constructed with the intention of emphasizing four factors: induction, space, perceptual speed, and deduction.

"Induction" items. For the six "induction" items, the distributions of loadings on the nine factors were as follows:

Factor	*Loading*			
	.30 or above	.20. to .29	.19 to —.19	—.20 to —.29
I	1	0	5	0
II	1	0	5	0
III	0	0	6	0
IV	0	0	6	0
V	1	0	5	0
VI	2	0	4	0
VII	0	1	5	0
VIII	1	1	4	0
IX	2	1	2	1

On only one of the factors (IX) were the loadings of half of these items greater than zero. Analysis of the loadings of all items on this factor revealed that there were five loadings of .30 and above and three loadings between .20 and .29. One could say dogmatically that the items heavily loaded with this factor involved more than did the other items the necessity of figuring out new principles and that success on these items depended more largely on ability to discover the principle than was true of the other items. But, in light of the small amount of information available about the items, such a statement could be nothing more than a judgment made to fit the findings to the desired results. No obvious common characteristics were present in the items with high loadings on this factor.

"Space" items. For the ten "space" items, the loadings were distributed as follows:

On all the factors, half or more of these items had vanishing or low negative loadings. Factor I had three loadings of .30 or above and two loadings between .20 and .29. Examination was made of all the items with loadings on this factor to see whether the factor might possibly be identified as spatial in nature. It seemed possible to think of all items with loadings of .20 and above as involving some sort of ability to deal with spatial relations. However, a large number of items with zero loadings in this factor also dealt with adjustments of figures in space. If the factor does involve spatial ability, it is evidently of some restricted sort. The exact nature of the function involved in this factor is not readily identified from examination of the items.

Factor	*Loading*			
	.30 or above	.20. to .29	.19 to —.19	—.20 to —.29
I	3	2	5	0
II	2	1	7	0
III	1	2	7	0
IV	1	2	6	1
V	1	2	7	0
VI	2	0	8	0
VII	1	1	8	0
VIII	3	1	5	1
IX	1	1	8	0

On Factor VIII, the "space" items had three loadings of .30 or above and one between .20 and .29. Because of the possibility that this factor might be spatial in nature, analysis was made of all the items with heavy loadings. Six items had loadings above .30 and three more had loadings between .20 and .29. These had no obvious characteristics in common. Moreover, some items which apparently involved similar tasks had zero or negative loadings. The identification of the factor, therefore, is not clear.

"Perceptual speed" items. For the nine "perceptual speed" items, the distribution of loadings on the various factors was as follows:

Of the nine factors, the one in which these items have the largest set of positive and significant loadings is Factor VII. However, in the case of this factor, one "perceptual speed" item has a negative loading large enough to be considered non-vanishing. When all the items with significant loading on this factor were considered, it was possible to see that success in answering them might have depended to a considerable extent on ability to pick out the proper answer from material involving complicated and detailed figures (the hypothesis used in setting up "perceptual speed" items). With the exception of item 3, this might also have been true of all the items with loadings between .20 and .29. The one "perceptual speed" item which has a negative loading for this factor, on inspection, appears to involve a rather unusual principle. If this observation is correct, it is possible that some other function was the prominent one in this item and that perceptual speed might have been an actual hindrance. This factor appears to be the most clear-cut of any found in the analysis of the figure-analogies material. The results do not fit those predicted by the hypotheses accurately, but there is sufficient similarity to justify further work with the method.

Factor	*Loading*				
	.30 or above	.20 to .29	.19 to —.19	—.20 to —.29	—.30 or above
I	0	0	8	1	0
II	2	1	6	0	0
III	2	1	6	0	0
IV	2	2	5	0	0
V	1	0	8	0	0
VI	1	1	7	0	0
VII	4	2	2	1	0
VIII	1	1	7	0	0
IX	0	1	7	0	1

That this factor came out as clearly as it did indicates that, with figure-analogies data, variance in perceptual

speed may reveal itself in terms of answers on specific items as well as in terms of number of items completed. This may result from the fact that individuals are not so familiar with this type of material as with number relations and that, therefore, the answers to figure-analogies items are more likely to be guessed than figured by slow, careful work. However, here again, as in the case of the number-series items, perceptual speed variance could probably be studied more effectively if the time limit on individual items were controlled.

"Deduction" items. The distribution of loadings for the five "deduction" items was as follows:

Factor	*Loading*				
	.30 or above	.20 to .29	.19 to —.19	—.20 to —.29	—.30 or below
I	0	1	4	0	0
II	0	1	4	0	0
III	0	0	5	0	0
IV	2	1	2	0	0
V	0	0	4	0	1
VI	1	1	3	0	0
VII	1	1	2	0	1
VIII	1	0	4	0	0
IX	2	0	2	0	1

Factor IV is the only one in which most of these items have positive loadings. When all the items loaded with this factor are considered, however, no evident similarities set them off from the items with vanishing loadings.

Other factors. Attempts to ascertain the nature of Factor II, Factor III, Factor V, and Factor VI (not heretofore considered), by studying the items with high loadings in each, were unsuccessful.

Summary and general discussion of results. For the figure-analogies items, nine factors were found. None of the four groups of items which had been set up according to Guilford's hypotheses was identified clearly with any one of these factors. Even for the best approach to such a situation (in the case of the "perceptual speed" items) one third of the items had vanishing or negative loadings.

Why results on this analysis should have been almost entirely negative, whereas on the number-series one they were largely positive is an interesting question. It is possible that the centroid axes were not rotated into positions that could be given meaningful psychological interpretation. However, the criteria set up by Thurstone were followed, and the final rotations were examined by an experienced factor analyst, who agreed that further rotations did not appear justified. The fact that the two analyses—both done by the same individual—came out with different types of result suggests that the differences may have resided in the nature of the items themselves. It seems probable that responses to mathematical items are more circumscribed (by the fact that we learn definite rules for handling numbers) than are responses to figure analogies (which are more foreign to our experience and which we have learned no definite method of handling). If this is the case, one would expect more meaningful factors from the number-series analysis because (1) it would be easier to determine in advance the processes used in solving number-series items, (2) there would be fewer processes used in solving them, with the resultant possibility of higher correlations between items, and (3) the processes used for solving them would probably be more easily identified because more familiar.

Another condition that may have caused the negative results in the figure-analogies test is the fact that the intercorrelations of the items were low and their probable errors, large. Of the 435 correlations, 412 might have occurred more than once in a hundred times by

chance, even if the actual correlations had been zero. Three hundred fifty-five of them might have occurred more than five in a hundred times by chance alone. Thus, a great deal of the variance accounted for by the factors may have been nothing but error variance. If this were the case, no meaningful interpretation of the factors would be expected. Because of the possibility that such was the case, further work on these items needs to be done with a very much larger number of subjects (say, 1000). Results for so many individuals would yield correlations with smaller probable errors and would provide considerably more assurance that the findings were not determined by chance alone.

Identification of factors in studies dealing with whole tests is aided by the fact that each test furnishes considerable information concerning the functions it involves. With individual items—and particularly with such similar ones as were used in this study—there is much less material to draw on for purposes of interpretation. It may well be that the factors found in the figure-analogies analysis have very clear meaning if one could get a statement of what each involves. Here again, the next step indicated is to get reports from a large number of individuals as to the procedures they follow in working with the items. Such accounts may point the way to identification of the factors found and, at the same time, furnish more definite ideas as to specific methods of constructing items.

C. RELATION OF FACTORS TO DIFFICULTY OF ITEMS

From work with factor analysis, Ferguson has drawn the conclusion that each degree of difficulty in a group of test items may appear as a separate factor (7). No exhaustive study of the relation of degree of item difficulty to the factors found has been made in this study. However, examination of the proportion of passes on each item in relation to size of factor loadings revealed that only one factor seemed to be closely connected with the difficulty of the items (Factor IV in the number-series items). This factor was identified as a number factor, and it was suggested that it would be expected to a greater degree in items with simple principles than in those involving more difficult rules. Whether the factor resulted merely from the difference in difficulty of the various items, or whether different functions were used in working with the items of different difficulty could not be decided from the available data.

Chapter V

Summary

The purpose of this study was to determine whether the factor loading of test items could be influenced by constructing them according to certain hypotheses regarding the nature of factors of human ability.

Number-series completion items and figure-analogies items were formulated according to suggestions by J. P. Guilford concerning the types of items which might be expected to have loadings in particular factors. (For example, three hypotheses were that the factor of spatial thinking would be most in evidence in figure-analogies items when the principle of change depended upon: a rotation, an inversion, or a rearrangement of parts.) With results from 300 university students, two factor analyses were made, using the Thurstone method: one of 17 number-series items (designed to emphasize induction, deduction, and number ability) and the other of 30 figure-analogies items (designed to emphasize perceptual speed, spatial ability, induction, and deduction).

Five factors were obtained in the analysis of the 17 number-series items. From examination of the items with heavy loadings on each factor, two of these were identified as factors of number ability and deduction. The loadings of the items designed to emphasize these two factors indicated that use of the hypotheses in constructing the items did give some control over their factor structure. The findings were not clear cut, but analysis of the discrepancies indicated that some of the difficulty was caused by the fact that items fitted the hypotheses for more than one factor. A third factor was tentatively identified as an induction factor. It gave some indication that construction of items to emphasize the importance of ability to deal with new principles would result in a grouping of items so designed. However, the exceptions to this tendency made it far from certain. For the two other factors, no identification seemed possible.

With the figure-analogies data, nine factors were obtained. One of them may possibly have involved the characteristics of the perceptual speed factor. The other factors contained no evident characteristics which could be used in identifying the processes they involved. In general these findings did not support the hypotheses which had been used in setting up the items. This result seemed to arise partly from the nature of the test material and partly from the difficulty of determining whether an item was an adequate expression of one, and only one, hypothesis.

BIBLIOGRAPHY

1. Balinsky, B. An analysis of the mental factors of various age groups from nine to sixty. *Genet. Psychol. Monogr.*, 1941, *23*, 191-234.
2. Blakey, R. A re-analysis of a test of the theory of two factors. *Psychometrika*, 1940, *5*, 121-136.
3. Blakey, R. A factor analysis of a non-verbal reasoning test. *Educ. & psychol. Measurement*, 1941, *1*, 187-198.
4. Chesire, L., Saffir, M., & Thurstone, L. L. Computing diagrams for the tetrachoric correlation coefficient. Chicago: Univ. Chicago Bookstore, 1933.
5. Coombs, C. A factorial study of number ability. *Psychometrika*, 1941, *6*, 161-189.
6. Coombs, C. A criterion for significant common factor variance. *Psychometrika*, 1941, *6*, 267-272.
7. Ferguson, G. The factorial interpretation of test difficulty. *Psychometrika*, 1941, *6*, 323-329.
8. Guilford, J. P. Psychometric methods. New York: McGraw-Hill, 1936.
9. Guilford, J. P. Human abilities. *Psychol. Rev.*, 1940, *47*, 367-394.
10. Guilford, J. P. & R. B. Personality factors *D, R, T,* and *A. J. abnorm. (soc.) Psychol.*, 1939, *34*, 21-36.
11. Guilford, J. P. & Lyons, T. On determining the reliability and significance of a tetrachoric coefficient of correlation. *Psychometrika*, 1942, *7*, 243-249.
12. Harrell, W. A factor analysis of mechanical ability tests. *Psychometrika*, 1940, *5*, 17-33.
13. Holzinger, K. & Harmon, H. Comparison of two factorial analyses. *Psychometrika*, 1938, *3*, 45-60.
14. Holzinger, K. & Swineford, F. A study in factor analysis: the stability of a bi-factor solution. *Supplementary educ. Monogr.*, 1939, *48*, 1-89.
15. Thurstone, L. L. Primary mental abilities. *Psychometr. Monogr.*, 1938, *1*, 1-121.
16. Thurstone, L. L. The perceptual factor. *Psychometrika*, 1938, *3*, 1-17.
17. Thurstone, L. L. Experimental study of simple structure. *Psychometrika*, 1940, *5*, 153-168.
18. Woodrow, H. The common factors in fifty-two mental tests. *Psychometrika*, 1939, *4*, 99-108.